Week 11 PYQs
Machine Learning Foundations
May 22 - Sep 25

Note

1. If no questions are listed in that term Quiz, it indicates that the corresponding quiz did not include questions from that week.

2. The answer key may contain minor inaccuracies, as answers were taken directly from the original quiz/exam papers. Some of these might have been corrected later.

3. For any issues or suggestions regarding this document, feel free to contact:
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Quiz 1 May 22

Quiz 2 May 22

ET May 22

1. Let X and Y be two random variables of an experiment such that $Y = 8X + 4$. If $E[Y] = 44$, what is the value of $E[7X + 10]$? *NOTE: Enter your answer to the nearest integer.* [Ans: 45]

2. Probability density function of a random variable X is given by

$$f(x) = \begin{cases} a|x| & \text{if } |x| \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

   Calculate the value of a. *NOTE: Enter your answer upto two decimal.* [Ans: 0.23 to 0.27]

3. For the probability density function $f(x)$ defined in the previous question, find the $P(X \geq \frac{1}{2})$. *NOTE: Enter your answer upto two decimal.* [Ans: 0.42 to 0.48]

4. Each set of a 3-set badminton match between P.V. Sindhu and Tai Tzu Ying is won with equal probability by either player. Let A be the event that all the three sets are won by one player. Let C be the event that at least two sets are won by Tai Tzu Ying, and let B denote the event that at most two sets are won by Tai Tzu Ying. Which of the two events are independent?

   (a) A and B are independent.

   (b) A and C are independent.

   (c) B and C are independent.

   (d) A, B and C are independent to each other.

   [Ans: 2]

5. Suppose bulb lifetimes follow an exponential distribution with parameter λ. Testing 5 bulbs yields lifetimes of 3, 4, 2, 5, and 6 years. What is the Maximum likelihood estimate for λ? *NOTE: Enter your answer upto two decimal.* [Ans: 0.23 to 0.27]

6. Given the transformation $U = X + Y$ and $V = X$, calculate the $|J|$, where J represents the Jacobian of the transform $(U, V) \mapsto (X, Y)$.

   (a) 1

   (b) 1/2

   (c) 0

   (d) U

   [Ans: 1]

7. Given $f(x,y) = \frac{1}{2}xe^{-y}$ for $0 < x < 2, y > 0$, find the joint distribution $g(u,v)$ under $U = X + Y$ and $V = X$.

   (a) $g(u,v) = \frac{1}{2}ve^{-(u-v)}$ if $0 < v < 2, u > v$, otherwise 0.

   (b) $g(u,v) = \frac{1}{2}ve^{-(u+v)}$ if $0 < v < 2, u > v$, otherwise 0.

   (c) $g(u,v) = \frac{1}{2}ve^{-(u-v)}$ if $0 < v < 2, v > u$, otherwise 0.

   (d) $g(u,v) = \frac{1}{2}ve^{-(u-v)}$ if $0 < x < 2, u > v$, otherwise 0.

[Ans: 1]

Quiz 1 Sep 22

Quiz 2 Sep 22

ET Sep 22

1. The heights (in cm) of a randomly selected learner of IITM are normally distributed with unknown mean μ and standard deviation σ. A random sample of 15 learners gave the following heights. 109, 143, 133, 160, 106, 152, 177, 161, 142, 124, 152, 148, 138, 129, 180 Based on the available information, find the maximum likelihood estimator of μ, the mean height of the learners. Enter the answer correct to two decimal places. [Ans: 142 to 146]

2. Let X be a uniformly continuous random variable on [0,100]. Pick a real number from [0, 100], call this number 'a'. A number will then drawn from X. The cost incurred in playing this game is as follows: $Cost = \begin{cases} 2(a-x), & if\ x \leq a \\ x-a, & if\ x > a \end{cases}$ What number should you pick to minimize the expected cost? Enter the answer correct to two decimal places. [Ans: 33 to 34]

3. The joint density function of two continuous random variables W and X be given by: $f_{WX}(w,x) = \begin{cases} 6w^2x, & 0 < w, x < 1 \\ 0, & otherwise \end{cases}$ Let $Y = W^2$, Z=WX. Find the joint distribution of Y and Z.

   (a) $g_{YZ}(y,z) = \begin{cases} \frac{3z}{\sqrt{y}}, & 0 < y < 1, 0 < z < 1 \\ 0, & otherwise \end{cases}$

   (b) $g_{YZ}(y,z) = \begin{cases} -\frac{3z}{\sqrt{y}}, & w^2 < y < 1, 0 < z < 1 \\ 0, & otherwise \end{cases}$

   (c) $g_{YZ}(y,z) = \begin{cases} \frac{3z}{\sqrt{y}}, & w^2 < y < 1, 0 < z < 1 \\ 0, & otherwise \end{cases}$

   (d) $g_{YZ}(y,z) = \begin{cases} 6\sqrt{y}z, & w^2 < y < 1, 0 < z < 1 \\ 0, & otherwise \end{cases}$

   [Ans: 3]

Quiz 1 Jan 23

Quiz 2 Jan 23

ET Jan 23

1. Consider two random variables X and Y with the following information. $E(X) = 2, E(Y) = 3$ $E(X^2) = 9, E(Y^2) = 11, E(XY) = 8$ Find $var(X + Y)$.

   (a) 7

   (b) 11

   (c) 10

   (d) 17

   (e) 3

   [Ans: 11]

2. The joint density function of random variables X and Y is given by $f_{XY}(x, y) = \begin{cases} (x^3 + y^3)/16, & 0 < x < 2, 0 < y < 1 \\ 0, & otherwise \end{cases}$, What is the value of $P(0 < X < 1, 0 < Y < 1)$

   (a) 1/16

   (b) 1/2

   (c) 1/12

   (d) 1/32

   [Ans: 1/32]

3. Let $X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \sim Normal(\mu, \Sigma)$, where $\mu = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$ and $\Sigma = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$. What will be the distribution of $Y = 2X_1 + X_2$?

   (a) $Y \sim Normal(20, 4)$

   (b) $Y \sim Normal(20, 8)$

   (c) $Y \sim Normal(20, 16)$

   (d) $Y \sim Normal(20, 32)$

   [Ans: Normal(20, 16)]

Quiz 1 May 23

Quiz 2 May 23

ET May 23

1. Suppose a firm produces X number of equipments on day i, where $i : 1, 2, ..., n$. Each of the X_i's are independent and identically distributed with mean 4 and variance 9. Using CLT, find the approximate probability that the total number of equipments produced in 100 days is greater than 450. Enter the answer correct to three decimal places. Hint: Use the following values of F_Z if required: $F_Z(-1.66) = 0.04846$ $F_Z(-3.2) = 0.00064$ $F_Z(1.66) = 0.95154$ $F_Z(3.2) = 0.99936$ [Ans: 0.044 to 0.052]

2. Suppose a fair coin is tossed twice. Let A denote the event that the first toss is a tail. Let B denote the event that the second toss is a tail. Let C denote the event that out of the first two tosses, either both are heads or both are tails. Which among the following options are true? Select all that apply.

   (a) A, B and C are independent.

   (b) A, B and C are dependent.

   (c) A, B and C are pairwise independent.

   (d) $P(A) = P(B) = P(C) = 1/2$

   [Ans: 2, 3, 4]

3. Let X and Y have the following joint probability density function:
   $f_{XY}(x, y) = \begin{cases} 3/2, & x > 0, x^2 < y < 1 \\ 0, & otherwise \end{cases}$ Based on the above data, answer the given subquestions. Are X and Y independent?

   (a) Yes

   (b) No

   [Ans: 2]

4. Find $Pr(X > Y)$ Enter the answer correct to two decimal places. [Ans: 0.25]

Quiz 1 Sep 23

Quiz 2 Sep 23

ET Sep 23

1. Let X and Y be two independent random variables, where X Bernoulli $(\frac{1}{2})$ and Y Bernoulli $(\frac{1}{4})$. Determine $P(X+Y=1)$ and enter the answer correct to one decimal place. [Ans: 0.5]

2. Let X and Y have the following joint density function $f(x,y) = \begin{cases} x(y-x)e^{-y}, & 0 < x \leq y < \infty \\ 0, & otherwise \end{cases}$.
   Find the conditional expectation $E(X|Y=2)$. [Ans: 1]

Quiz 1 Jan 24

Quiz 2 Jan 24

ET Jan 24

1. Let X_1 and X_2 be independent exponential random variables with parameters λ_1 and λ_2. Let the joint probility density function of X_1 and X_2 be $f_{X_1X_2}$. Define $Y_1 = X_1+X_2$ and $Y_2 = X_1 - X_2$. Which of the following represents the joint density function of Y_1 and Y_2 in terms of $f_{X_1X_2}$? Note: The probability density function of an exponential random variable (X) with parameter λ is given by, $\lambda e^{-\lambda x}$.

   (a) $f_{Y_1Y_2}(y_1, y_2) = \begin{cases} \frac{\lambda_1\lambda_2}{2} e^{\{-\lambda_1(\frac{y_1+y_2}{2})-\lambda_2(\frac{y_1-y_2}{2})\}} & y_1 + y_2 \geq 0, y_1 - y_2 \geq 0 \\ 0 & \text{otherwise} \end{cases}$

   (b) $f_{Y_1Y_2}(y_1, y_2) = \begin{cases} \frac{\lambda_1\lambda_2}{2} e^{\{-\lambda_1(\frac{x_1+x_2}{2})-\lambda_2(\frac{x_1-x_2}{2})\}} & y_1 + y_2 \geq 0, y_1 - y_2 \geq 0 \\ 0 & \text{otherwise} \end{cases}$

   (c) $f_{Y_1Y_2}(y_1, y_2) = \begin{cases} \frac{\lambda_1\lambda_2}{2} e^{\{-\lambda_1(\frac{y_1-y_2}{2})-\lambda_2(\frac{y_1+y_2}{2})\}} & y_1 + y_2 \geq 0, y_1 - y_2 \geq 0 \\ 0 & \text{otherwise} \end{cases}$

   (d) $f_{Y_1Y_2}(y_1, y_2) = \begin{cases} \frac{\lambda_1\lambda_2}{2} e^{\{-\lambda_1(\frac{y_1+y_2}{2})-\lambda_2(\frac{y_1-y_2}{2})\}} & y_1 \geq 0, y_2 \geq 0 \\ 0 & \text{otherwise} \end{cases}$

   [Ans: 1]

2. The joint probability density function of X and Y is given by,

$$f(x, y) = \begin{cases} e^{-(x+y)} & 0 < x < \infty, 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

   Find the expected value of X i.e. $E[X]$. [Ans: 1]

3. In a large town, it is estimated that 20 [Ans: 64]

Quiz 1 May 24

Quiz 2 May 24

ET May 24

1. Let X and Y be two independent random variables, where $X \sim Normal(-1, 11)$ and $Y \sim Normal(1, 9)$. Define $U = 2X - 3Y$. Find the value of $P(U > 2)$. Enter the answer correct to three decimal places. Hint: Use the following values of F_Z if required. F_Z stands for the CDF of the standard normal.

   - $F_Z(0.62) = 0.7343$
   - $F_Z(-0.62) = 0.2656$
   - $F_Z(1.25) = 0.8947$
   - $F_Z(-1.25) = 0.1052$

   [Ans: 0.262 to 0.268]

2. Let X and Y have the following joint density function

$$f(x,y) = \begin{cases} 18x^2y^2 & x, y \geq 0, x + y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   Based on the above data, answer the given subquestions.

   (a) Find the conditional distribution $f_{X|Y}(x|y)$.

   i. $f_{X|Y}(x|y) = \frac{32}{(1-y)^3}, 0 < x < 1$
   ii. $f_{X|Y}(x|y) = \frac{3x^2}{(1-y)^3}, 0 < x < 1 - y$
   iii. $f_{X|Y}(x|y) = 3x^2(1-y)^3, 0 < x < 1$
   iv. $f_{X|Y}(x|y) = 3x^2(1-y)^3, 0 < x < 1 - y$

   [Ans: 2]

   (b) Find the value of $P(X < \frac{1}{2}|Y = \frac{1}{2})$. [Ans: 1]

Quiz 1 Sep 24

Quiz 2 Sep 24

ET Sep 24

1. Suppose X and Y are independent random variables, with $X \sim Uniform(0,1)$ and $Y \sim Uniform(0,1)$. Define $Z = \min(X,Y)$. What is the cumulative distribution function $F_Z(z)$ for Z?

   (a) $F_Z(z) = z$ for $0 \leq z \leq 1$

   (b) $F_z(z) = z^2$, for $0 \leq z \leq 2$

   (c) $F_z(z) = 2z - z^2$ for $0 \leq z \leq 2$

   (d) $F_Z(z) = 1 - (1-z)^2$, for $0 \leq z \leq 1$

   [Ans: 4]

2. Let X and Y have the joint pdf $f_{X,Y}(x,y) = \frac{1}{27}(X+Y)$ for $0 \leq X \leq 3$ and $0 \leq Y \leq 3$. Define $U = X + Y$ and $V = X - Y$. What is the joint pdf of U and V?

   (a) $f_{UV}(u,v) = \frac{u}{54}$ if $0 \leq u \leq 6$ and $-u \leq v \leq u$, 0 otherwise.

   (b) $f_{UV}(u,v) = \frac{u}{54}$ if $0 \leq u \leq 3$ and $-u \leq v \leq u$, 0 otherwise.

   (c) $f_{UV}(u,v) = \frac{u}{27}$ if $0 \leq u \leq 6$ and $-u \leq v \leq u$, 0 otherwise.

   (d) $f_{UV}(u,v) = \frac{v}{54}$ if $0 \leq v \leq 6$ and $-v \leq u \leq v$, 0 otherwise.

   [Ans: 1]

3. Let X and Y be independent random variables, where $X \sim Normal(0,1)$ and $Y \sim Normal(1,4)$. Define $W = 3X - 2Y$. What is the distribution of W?

   (a) Normal(-2, 7)

   (b) Normal(-2, 13)

   (c) Normal(-2, 25)

   (d) None of these

   [Ans: 3]

Quiz 1 Jan 25

Quiz 2 Jan 25

ET Jan 25

1. Let X be an exponentially distributed random variable with parameter $\lambda = 1$. What is the CDF (cumulative distribution function), $F_X(5)$?

   (a) $F_X(5) = 1 - e^{-5}$

   (b) $F_X(5) = 1 + e^{-5}$

   (c) $F_X(5) = e^{-5}$

   (d) $F_X(5) = e^{-5} - 1$

   [Ans: 1]

2. The random variable y is being transformed according to the measurement mapping: $z = Cy + \epsilon$ where: $z \in \mathbb{R}^d$, $C \in \mathbb{R}^{d \times n}$, and $\epsilon \sim \mathcal{N}(0, \sigma^2 I_d)$ is independent Gaussian (measurement) noise. Which of the following represents the conditional probability distribution $p(z|y)$?

   (a) $p(z|y) = \mathcal{N}(Cy, \sigma^2 I_d)$

   (b) $p(z|y) = \mathcal{N}(Cy, C^T \sigma^2 I_d C)$

   (c) $p(z|y) = \mathcal{N}(y, \sigma^2 I_d)$

   (d) $p(z|y) = \mathcal{N}(Cy, \sigma^2 C C^T)$

   [Ans: 1]

3. Let $X_1 \sim N(2, 1)$ and $X_2 \sim Poisson(3)$ be independent random variables. What is the expectation of the compound random variable $Y = 3X_1 + 2X_2$? [Ans: 12]

Quiz 1 May 25

Quiz 2 May 25

ET May 25

1. Four friends, Priya, Rahul, Sonia, and Tara, are randomly assigned to four distinct chairs in a row at a concert. Let event C be that Sonia and Tara are assigned to chairs that are next to each other (in either order), and let event D be that Priya is assigned to a chair that is not at either end. Find the conditional probability, $P(C|D)$ Enter the answer correct to two decimal places. [Ans: 0.31 to 0.37]

2. The joint probability density function of random variables X and Y is given by:

$$f(x,y) = \begin{cases} 24xy, & \text{for } 0 < x < 1, 0 < y < 1, \text{ and } x + y < 1 \\ 0, & \text{otherwise} \end{cases}$$

   Find the variance of Y, i.e., compute $Var(Y)$. Enter the answer correct to two decimal places. [Ans: 0.02 to 0.06]

3. Let X Normal(3,4) and Y Normal(-2,5) be two independent random variables. Define the new variable $U = 4X + Y$. Find the value of $P(U < 10)$. Hint: Use the following values of F_Z, the CDF of the standard normal distribution:

   - $F_Z(0.89) = 0.8133$
   - $F_Z(-0.89) = 0.1867$
   - $F_Z(1.38) = 0.9162$
   - $F_Z(-1.38) = 0.0838$
   - $F_Z(0) = 0.5$

   [Ans: 0.5]

Quiz 1 Sep 25

Quiz 2 Sep 25

ET Sep 25